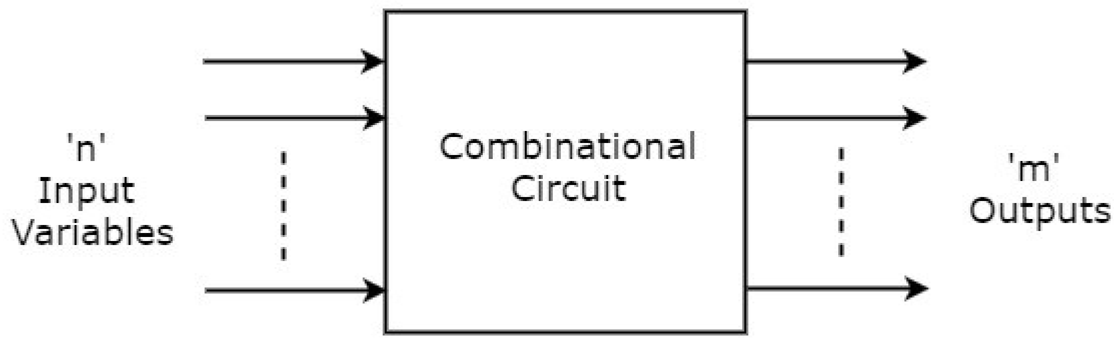
Arithmetic Circuits

Combinational circuits consist of Logic gates. These circuits operate with binary values. The outputss of combinational circuit depends on the combination of present inputs. The following figure shows the **block diagram** of combinational circuit.



This combinational circuit has ‘n’ input variables and ‘m’ outputs. Each combination of input variables will affect the outputss.

Combinational circuit does not have memory so called memory less device.

Design procedure of Combinational circuits

- Find the required number of input variables and outputs from given specifications.
- Formulate the **Truth table**. If there are ‘n’ input variables, then there will be 2n possible combinations. For each combination of input, find the output values.
- Find the **Boolean expressions** for each output. If necessary, simplify those expressions.
- Implement the above Boolean expressions corresponding to each output by using **Logic gates**.

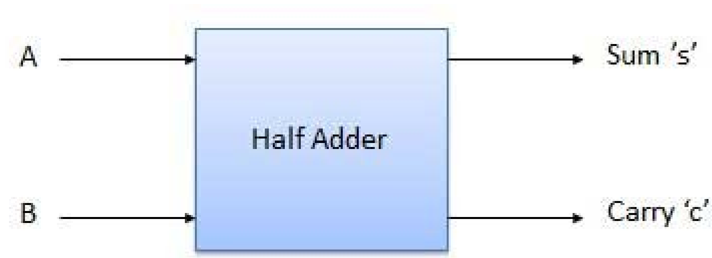
Binary Adder

The most basic arithmetic operation is addition. The circuit, which performs the addition of two binary numbers, is known as **Binary adder**. First, let us implement an adder, which performs the addition of two bits.

Half Adder

Half adder is a combinational logic circuit with two inputs and two outputs. The half adder circuit is designed to add two single bit binary number A and B. It is the basic building block for addition of two **single** bit numbers. This circuit has two outputs **carry** and **sum**.

Block diagram



The **Truth table** of Half adder is shown below.

Inputs		**Outputs**	
A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

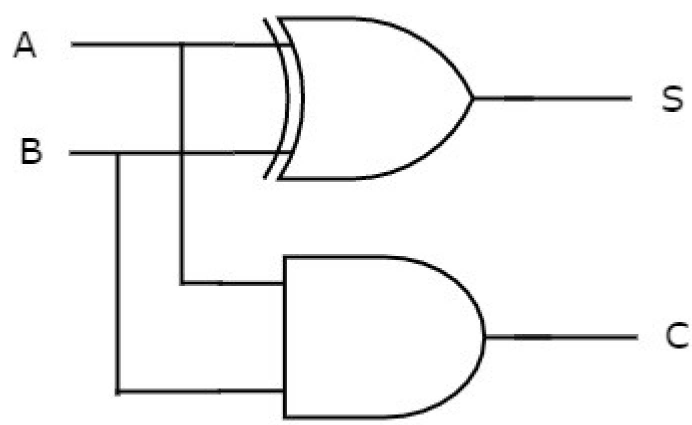
Sum S is the Least significant bit and carry, C is the Most significant bit of the resultant sum. For first three combinations of inputs, carry, C is zero and the value of S will be either zero or one based on the **number of ones** present at the inputs. But, for last combination of inputs, carry, C is one and sum, S is zero, since the resultant sum is two.

From Truth table, we can directly write the **Boolean functions** for each output as

$$S=A\oplus B$$

$$C=AB$$

We can implement the above functions with 2-input Ex-OR gate & 2-input AND gate. The **circuit diagram** of Half adder is shown in the following figure.

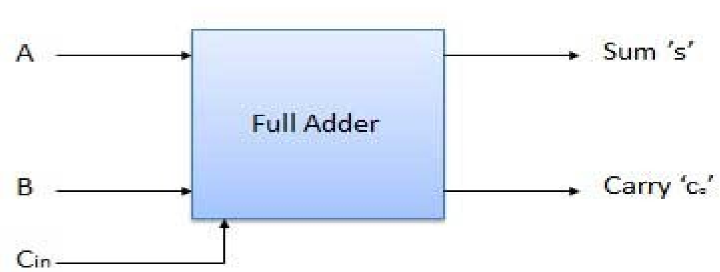


In the above circuit, a two input Ex-OR gate & two input AND gate produces sum, S & carry, C respectively. Therefore, Half-adder performs the addition of two bits.

Full Adder

Full adder is a combinational logic circuit with three inputs and two outputs Full adder is developed to overcome the drawback of Half Adder circuit. It can add two one-bit numbers A and B, and carry c. The full adder is a three input and two output combinational circuit.

Block diagram



The **Truth table** of Full adder is shown below.

Inputs			Outputs	
A	**B**	**C_{in}**	**C_{out}**	**S**
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

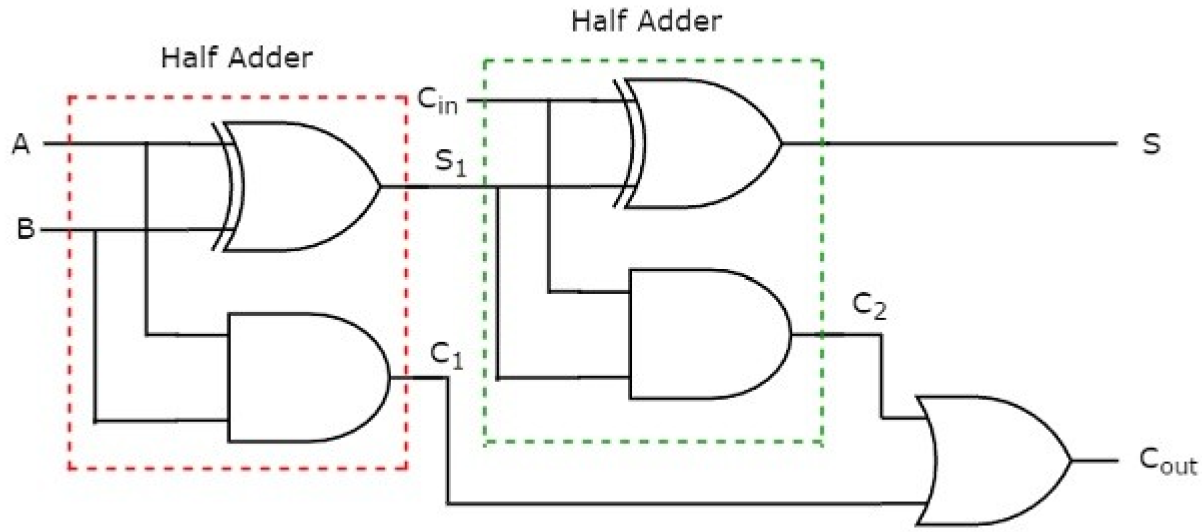
sum, S is the Least significant bit and carry, C_{out} is the Most significant bit of resultant sum. It is easy to fill the values of outputs for all combinations of inputs in the truth table. Just count the **number of ones** present at the inputs and write the equivalent binary number at outputs. If C_{in} is equal to zero, then Full adder truth table is same as that of Half adder truth table.

We will get the following **Boolean functions** for each output after simplification.

$$S=A\oplus B\oplus Cin$$

$$cout=AB+(A\oplus B)cin$$

The sum, S is equal to one, when odd number of ones present at the inputs. We know that Ex-OR gate produces an output, which is an odd function. So, we can use either two 2input Ex-OR gates or one 3-input Ex-OR gate in order to produce sum, S. We can implement carry, C_{out} using two 2-input AND gates & one OR gate. The **circuit diagram** of Full adder is shown in the following figure.



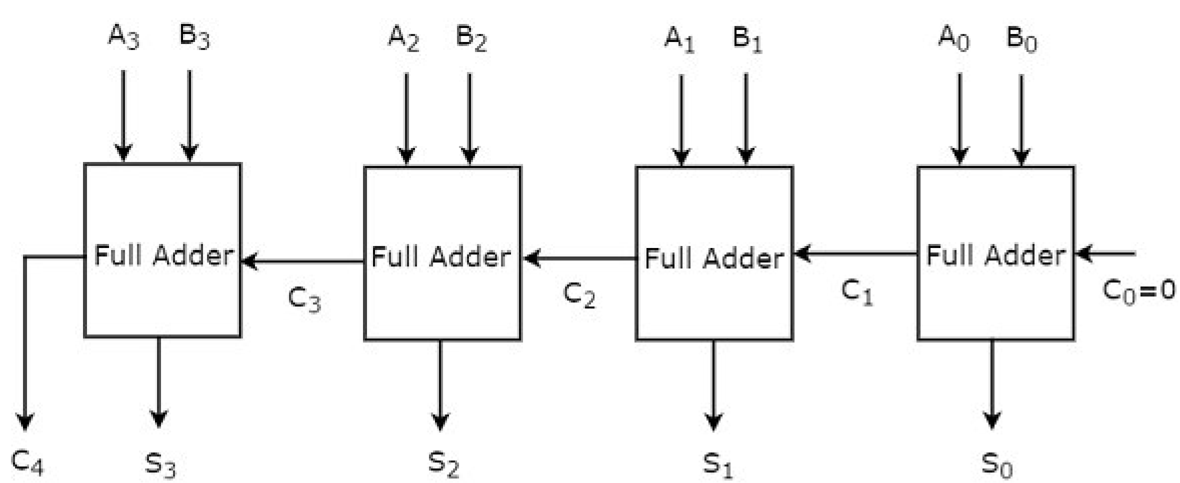
This adder is called as **Full adder** because for implementing one Full adder, we require two Half adders and one OR gate. If C_{in} is zero, then Full adder becomes Half adder. We can verify it easily from the above circuit diagram or from the Boolean functions of outputs of Full adder.

4-bit Binary Adder

The 4-bit binary adder performs the **addition of two 4-bit numbers**. Let the 4-bit binary numbers, A=A3A2A1A0 and B=B3B2B1B0. We can implement 4-bit binary adder in one of the two following ways.

- Use one Half adder for doing the addition of two Least significant bits and three Full adders for doing the addition of three higher significant bits.
- Use four Full adders for uniformity. Since, initial carry C_{in} is zero, the Full adder which is used for adding the least significant bits becomes Half adder.

For the time being, we considered second approach. The **block diagram** of 4-bit binary adder is shown in the following figure.

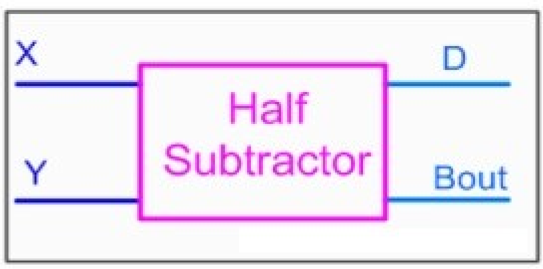


Here, the 4 Full adders are cascaded. Each Full adder is getting the respective bits of two parallel inputs A & B. The carry output of one Full adder will be the carry input of subsequent higher order Full adder. This 4-bit binary adder produces the resultant sum having at most 5 bits. So, carry out of last stage Full adder will be the MSB.

In this way, we can implement any higher order binary adder just by cascading the required number of Full adders. This binary adder is also called as **ripple carry** binarybinary **adder** because the carry propagates ripplesripples from one stage to the next stage

Half Subtractors

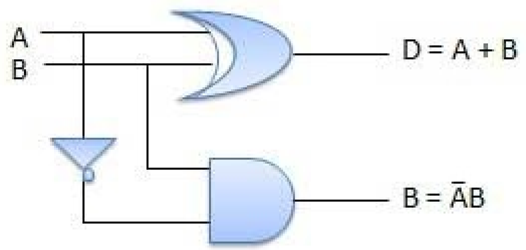
Half subtractor is a combination circuit with two inputs and two outputs (difference and borrow). It produces the difference between the two binary bits at the input and also produces an output (Borrow) to indicate if a 1 has been borrowed. In the subtraction (A-B), A is called as Minuend bit and B is called as Subtrahend bit.



Truth Table

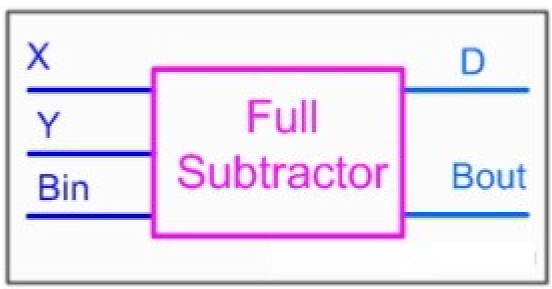
Inputs		Output	
A	B	(A – B)	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Circuit Diagram



Full Subtractors

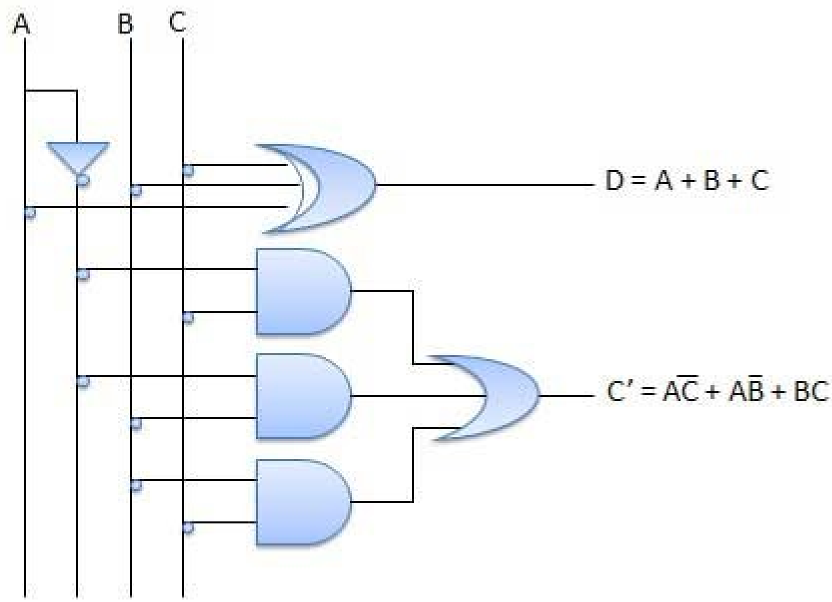
The disadvantage of a half subtractor is overcome by full subtractor. The full subtractor is a combinational circuit with three inputs A,B,C and two output D and C'. A is the 'minuend', B is 'subtrahend', C is the 'borrow' produced by the previous stage, D is the difference output and C' is the borrow output.



Truth Table

Inputs			Output	
A	B	C	(A-B-C)	C'
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Circuit Diagram



Binary Adder / Subtractor

The circuit, which can be used to perform either addition or subtraction of two binary numbers at any time is known as **Binary Adder / subtractor**. Both, Binary adder and Binary subtractor contain a set of Full adders, which are cascaded. The input bits of binary number A are directly applied in both Binary adder and Binary subtractor.

There are two differences in the inputs of Full adders that are present in Binary adder and Binary subtractor.

- The input bits of binary number B are directly applied to Full adders in Binary adder, whereas the complemented bits of binary number B are applied to Full adders in Binary subtractor.
- The initial carry, $C_0 = 0$ is applied in 4-bit Binary adder, whereas the initial carry borrowborrow, $C_0 = 1$ is applied in 4-bit Binary subtractor.

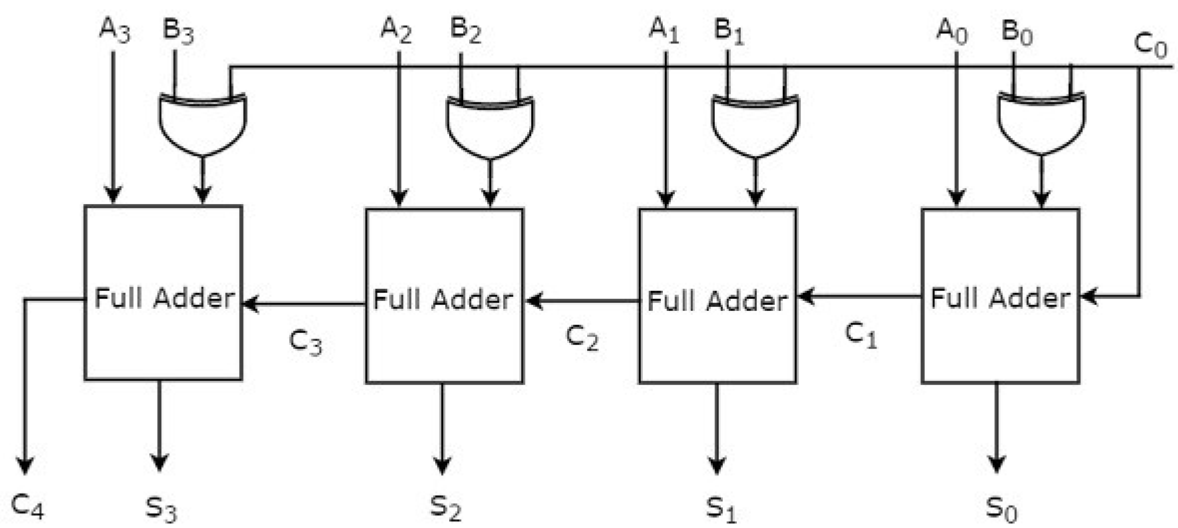
We know that a **2-input Ex-OR gate** produces an output, which is same as that of first input when other input is zero. Similarly, it produces an output, which is complement of first input when other input is one.

Therefore, we can apply the input bits of binary number B, to 2-input Ex-OR gates. The other input to all these Ex-OR gates is C_0. So, based on the value of C_0, the Ex-OR gates produce either the normal or complemented bits of binary number B.

4-bit Binary Adder / Subtractor

The 4-bit binary adder / subtractor produces either the addition or the subtraction of two 4-bit numbers based on the value of initial carry or borrow, C_0. Let the 4-bit binary numbers, A=A3A2A1A0 and B=B3B2B1B0. The operation of 4-bit Binary adder / subtractor is similar to that of 4-bit Binary adder and 4-bit Binary subtractor.

Apply the normal bits of binary numbers A and B & initial carry or borrow, C_0 from externally to a 4-bit binary adder. The **block diagram** of 4-bit binary adder / subtractor is shown in the following figure.



If initial carry, C_0 is zero, then each full adder gets the normal bits of binary numbers A & B. So, the 4-bit binary adder / subtractor produces an output, which is the **addition of two binary numbers** A & B.

If initial borrow, C_0 is one, then each full adder gets the normal bits of binary number A & complemented bits of binary number B. So, the 4-bit binary adder / subtractor produces an output, which is the **subtraction of two binary numbers** A & B.

Therefore, with the help of additional Ex-OR gates, the same circuit can be used for both addition and subtraction of two binary numbers.